Interest Rates

Gross (% per annum)

This is the rate of ***simple*** interest earned in a year (before deducting tax).
Dividing by 12 gives the monthly rate of interest.

Annual Equivalent Rate (%)

The AER gives the total annual interest (as a percentage) assuming that the initial deposit and all interest earned is left in the account for a full 12 months.
This is the rate that you should use to make comparisons between different accounts.

Net (% per annum)

Net interest is gross interest minus tax.
If tax is deducted at a rate of 20%, net interest is 80% of gross interest.
Non-taxpayers may register for payment of gross interest.

Countrywide Building Society

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grow

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Savings Growth Data Sheet

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Growth Savings Accounts

The tables give the rates of interest for our three high growth savings accounts. (correct to 2 decimal places).

The Annual Growth Account

Annual			
Amount £	**AER %**	**Gross % pa**	**Net % pa**
1 – 999	5.75	5.75	4.60
1000+	5.95	5.95	4.76

Interest is earned each day and added to the account once a year at close of business on 31st December.

The Quarterly Growth Account

Quarterly			
Amount £	**AER %**	**Gross % pa**	**Net % pa**
1 – 2999	5.72	5.60	4.48
3000+	6.01	5.88	4.70

Interest is earned each day and added to the account four times a year at close of business on 31st March, 30th June, 30th September and 31st December.

The Monthly Growth Account

Monthly			
Amount £	**AER %**	**Gross % pa**	**Net % pa**
1 – 1999	5.85	5.70	4.56
2000+	6.10	5.94	4.75

Interest is earned each day and added to the account at close of business on the last day of each month.

General Notes

In each account interest is earned on the current balance every day until the balance is next up-dated. If you withdraw money part of the way through the period between balance up-dates, interest will be paid on the previous balance up to and including the day prior to the withdrawal. Interest will be paid on the new balance from the day of withdrawal until the next balance up-date. For further information about these and our other accounts, visit one of our branches or our website.

The example which follows shows how the balance of an account increases for money left for a year in a Monthly Growth Account.

Example Monthly Growth Account

A saver deposits £500 on 1st January in the Monthly Growth Account.

The way in which these savings grow over one year is shown in the table below.

Date	Balance
1st Jan	**£500.00**
1st Feb	£502.38
1st Mar	£504.76
1st Apr	£507.16
1st May	£509.57
1st Jun	£511.99
1st Jul	£514.42
1st Aug	£516.86
1st Sep	£519.32
1st Oct	£521.79
1st Nov	£524.26
1st Dec	£526.75
1st Jan	**£529.26**
Total Interest	**£29.26**

N.B. All calculations have been done accurately. Values given in the table are rounded to 2 decimal places.

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Savings Growth Worksheet

The leaflet gives the interest rates for three different building society savings accounts. Read the information first, including the definitions of gross % rate, Annual Equivalent Rate (AER) and net % rate given on the back of the leaflet.

Net % Rates

The net % is the rate of interest after 20% tax is deducted. So the net % rate is 80% of the gross % rate.

Example to find the net % rate

For the Annual Growth Account with £1 – £999

Net rate $= 0.8 \times 0.0575$

$= 0.046 = 4.60\%$

For the Annual Growth Account with £1000+

Net rate $= 0.8 \times 0.0595$

$= 0.0476 = 4.76\%$

Complete the table to give the net % rates for the other savings accounts then check your answers with those given in the leaflet.

Annual		
Amount £	**Gross % pa**	**Net % pa**
1 – 999	5.75	4.60
1000+	5.95	4.76
Quarterly		
Amount £	**Gross % pa**	**Net % pa**
1 – 2999	5.60	
3000+	5.88	
Monthly		
Amount £	**Gross % pa**	**Net % pa**
1 – 1999	5.70	
2000+	5.94	

Annual Equivalent Rate

If you have a sum of money to invest for a number of years, the best rate to compare is the Annual Equivalent Rate (AER) i.e. the annual rate of interest you will get on your savings. For the Annual Growth Account this is the same as the gross % rate, but for the other accounts in which interest is added more than once per year, the AER is higher than the gross % rate because the interest is compounded.

You can check the table on the leaflet that shows how a deposit of £500 in the **Monthly Growth Account** grows over the course of a year by working as follows:

Monthly % interest = Gross % interest ÷ 12 = 5.7% ÷ 12 = 0.475%

If the balance at the end of the n^{th} month is £P_n,
the balance at the end of the $(n + 1)^{\text{th}}$ month, $P_{n+1} = P_n + 0.00475P_n = 1.00475P_n$

The relationship $P_{n+1} = 1.00475P_n$ is called a **recurrence relation.**
Given a starting value, P_0, the recurrence relation can be used over and over again to find the subsequent values P_1, P_2, P_3, …. in the sequence.

In this case the 'starting value' is the balance at the beginning i.e. £500.

Using this as P_0 gives $P_1 = 1.00475 \times 500 = 502.375$,
$P_2 = 1.00475 \times 502.375 = 504.7612813$ and so on.

Such calculations can be done quickly using a graphic calculator or spreadsheet.

Using a graphic calculator

- Enter the value of P_0, in this case 500, into your calculator.
- Now enter the recurrence relation using the Ans key as P_n i.e. 1.00475*Ans*

Repeatedly press the 'equals' key to give successive terms in the sequence.
Check that the values you get agree with those given in the table on the leaflet.

Finally use AER = $\dfrac{\text{Total Interest}}{\text{Balance at start of year}} \times 100$ to check the value given (i.e. 5.85%)

Using a spreadsheet

The formulae used in a spreadsheet to give the last table in the leaflet are shown below. Can you explain the formulae used in cells B7, F3, F15 and B4?

	A	B	C	D	E	F
1	**Monthly Growth Account**				**Date**	**Balance**
2					1st Jan	**500**
3	**Gross (% pa)**	5.7			1st Feb	=(1+$B$7/100)*F2
4	**AER**	=100*F15/F2			1st Mar	=(1+$B$7/100)*F3
5					1st Apr	=(1+$B$7/100)*F4
6	**Monthly**				1st May	=(1+$B$7/100)*F5
7	**% Rate**	=$B$3/12			1st Jun	=(1+$B$7/100)*F6
8					1st Jul	=(1+$B$7/100)*F7
9					1st Aug	=(1+$B$7/100)*F8
10					1st Sep	=(1+$B$7/100)*F9
11					1st Oct	=(1+$B$7/100)*F10
12					1st Nov	=(1+$B$7/100)*F11
13					1st Dec	=(1+$B$7/100)*F12
14					1st Jan	**=(1+$B$7/100)*F13**
15					**Total Interest**	**=F14-F2**

Copy these formulae onto a spreadsheet, using 'Fill down' for cells F4 to F14.
Check whether the resulting values match those given in the table in the leaflet.

By changing the values in cells B3 and F2, use your spreadsheet to give the monthly balances when £2500 is invested in the Monthly Growth Account for one year.
Does the AER given on your spreadsheet match the 6.10% suggested in the leaflet?

Print out your worksheet for £2500, then use a recurrence relation on your graphic calculator to check the monthly balances on your worksheet.
Also use the formula given above to check the value of the AER.

In the **Quarterly Growth Account**, a quarter of the gross profit is added at the end of each quarter (i.e. after every 3 months).

On a new worksheet, starting with a balance of £2000, use spreadsheet formulae to calculate:

- the quarterly % rate i.e. the % interest that is paid after every 3 months
- the amount in the account on 1st April, 1st July, 1st October and the beginning of the following year
- the total interest earned during the year
- the AER

Print your worksheet.
Use a graphic calculator to check the balances on your worksheet.
Also check that your value for the AER agrees with that given in the leaflet.

Now change the values in some of the cells on your worksheet so that it gives the balances and AER for an investment of £4000 in the Quarterly Growth Account.
Print your worksheet.
Check your values with a graphic calculator and the AER value given in the leaflet.

Future Value and AER Formulae

Future Value:
When a sum of money is invested for n time periods, its future value, FV, at the end of this time is given by:

$$FV = PV\,(1 + r)^n$$

where PV is the present value and r is the interest rate for one period (as a decimal)

AER:

The annual effective interest rate, r, given a nominal interest rate, i, is given by:

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$ where n is the number of compounding periods per year.

The above formulae provide a quick way to check particular balances and the AER.

Example to check the AER
Suppose we want to check the AER value given on the leaflet for an amount less than £2000 in the Monthly Growth Account.

The gross % interest per annum for an amount below £2000 is 5.7% so the value used for i is 0.057

The formula for AER gives: $r = \left(1 + \frac{0.057}{12}\right)^{12} - 1$

$= 1.00475^{12} - 1 = 1.05851\ldots - 1 = 0.05851\ldots$

The AER is 5.85% to 2 decimal places.

Example to check a Future Value

Suppose we wish to check the balance on 1st September that is given in the example on the leaflet for the Monthly Growth Account. This can be done using the Future Value formula, $FV = PV(1 + r)^n$

In this case:
- the present value PV is 500,
- the interest rate for one month is 5.7% ÷ 12 = 0.475% so $r = 0.00475$ (as a decimal)
- the number of months that have passed, $n = 8$

So $FV = PV(1 + r)^n$
$= 500(1 + 0.00475)^8$
$= 500 \times 1.00475^8 = 519.3188\ldots$ = £519.32 to the nearest pence.

Use the Future Value and AER formulae to check the final balances and the AER values given on each of your spreadsheet print-outs.

Investing for more than one year

To find the amount in any account at the end of n years, you should use the future value formula with the decimal value for the AER as the value of r.

For example, suppose you have £1000 that you can leave in any of the three accounts for a period of 10 years.

If you put £1000 in the Monthly Growth Account the AER is 5.85% (i.e. 0.0585)

The amount you would have at the end of 10 years is:

$FV = PV(1 + r)^n = 1000(1 + 0.0585)^{10} = 1000 \times 1.0585^{10}$
$= 1765.6662\ldots$ = £1765.67

The interest earned would be £765.67

Use the Future Value formula to find how much you would have in each of the other accounts at the end of 10 years and the interest earned.

Use a graphic calculator or spreadsheet to produce a table that shows the balance that would be in each account at the end of each year and check that the final amount (at the end of 10 years) agree with your answers using the Future Value formula.

Which account gives the most interest for £1000 invested for 10 years?

If you have used a spreadsheet for this comparison, you could use it to compare the interest on other amounts and invested for different time periods.

Savings Growth — Teacher Notes

Which Free-Standing Unit does this material support?
Advanced – Mathematical Principles of Personal Finance

What students need to know (before attempting this activity)

- How to calculate simple and compound interest.
- How to use spreadsheets, including formulae, absolute and relative referencing.

General Notes
The data sheet should be printed double-sided onto a single sheet and then folded to form a leaflet. The worksheet shows learners how to use a graphic calculator and/or spreadsheet to work out the interest on different amounts of money. It also includes use of formulae for future values and the AER .

Extension
The internet provides a good source of information about different ways of investing money and websites such as www.moneyfacts.com give up-to-date information and comparisons. In a follow-up activity learners could be asked to use the internet to find the savings account that gives the best return on a particular sum of money (eg £5000) that they could invest for a given time period (eg 5 years).
When discussing the results of this activity you could also discuss with learners the many complications that can arise when using savings accounts.

Answers

Page 1 Net % Rates

Annual		
Amount £	**Gross % pa**	**Net % pa**
1 – 999	5.75	4.60
1000+	5.95	4.76
Quarterly		
Amount £	**Gross % pa**	**Net % pa**
1 – 2999	5.60	4.48
3000+	5.88	4.70
Monthly		
Amount £	**Gross % pa**	**Net % pa**
1 – 1999	5.70	4.56
2000+	5.94	4.75

Page 2 £2500 invested in the Monthly Growth Account for 1 year

Monthly Growth Account		**Date**	**Balance**
		1st Jan	**£2,500.00**
Gross (% pa)	5.94	1st Feb	£2,512.38
AER	6.10	1st Mar	£2,524.81
		1st Apr	£2,537.31
Monthly		1st May	£2,549.87
% Rate	0.495	1st Jun	£2,562.49
		1st Jul	£2,575.17
		1st Aug	£2,587.92
		1st Sep	£2,600.73
		1st Oct	£2,613.61
		1st Nov	£2,626.54
		1st Dec	£2,639.54
		1st Jan	**£2,652.61**
		Total Interest	**£152.61**

Page 3 £2000 invested in the Quarterly Monthly Growth account for 1 year

Quarterly Growth Account		Date	Balance
		1st Jan	**£2,000.00**
Gross (% pa)	5.60	1st Apr	£2,028.00
AER	5.72	1st Jul	£2,056.39
		1st Oct	£2,085.18
Quarterly		1st Jan	£2,114.37
Quarterly % Rate	1.40	**Total Interest**	£114.37

Page 3 £4000 invested in the Quarterly Monthly Growth account for 1 year

Quarterly Growth Account		Date	Balance
		1st Jan	**£4,000.00**
Gross (% pa)	5.88	1st Apr	£4,058.80
AER	6.01	1st Jul	£4,118.46
		1st Oct	£4,179.01
		1st Jan	£4,240.44
Quarterly % Rate	1.47	**Total Interest**	£240.44

Page 4 Investing for more than one year

Amount at the end of 10 years in the Annual Growth Account
$= 1000\ (1 + 0.0595)^{10} = £1782.42$
Interest earned = £782.42

Amount at the end of 10 years in the Quarterly Growth Account
$= 1000\ (1 + 0.0572)^{10} = £1744.10$
Interest earned = £744.10

Account	Annual		Quarterly		Monthly	
Deposit	**£1000+**		**£1 - 2999**		**£1 - 1999**	
Year	Balance beginning	Balance end	Balance beginning	Balance end	Balance beginning	Balance end
1	£1,000.00	£1,059.50	£1,000.00	£1,057.20	£1,000.00	£1,058.50
2	£1,059.50	£1,122.54	£1,057.20	£1,117.67	£1,058.50	£1,120.42
3	£1,122.54	£1,189.33	£1,117.67	£1,181.60	£1,120.42	£1,185.97
4	£1,189.33	£1,260.10	£1,181.60	£1,249.19	£1,185.97	£1,255.35
5	£1,260.10	£1,335.07	£1,249.19	£1,320.64	£1,255.35	£1,328.78
6	£1,335.07	£1,414.51	£1,320.64	£1,396.18	£1,328.78	£1,406.52
7	£1,414.51	£1,498.67	£1,396.18	£1,476.05	£1,406.52	£1,488.80
8	£1,498.67	£1,587.84	£1,476.05	£1,560.48	£1,488.80	£1,575.89
9	£1,587.84	£1,682.32	£1,560.48	£1,649.74	£1,575.89	£1,668.08
10	£1,682.32	£1,782.42	£1,649.74	£1,744.10	£1,668.08	£1,765.67
Total Interest		**£782.42**		**£744.10**		**£765.67**

The Annual Growth Account gives the most interest on £1000 invested for 10 years.